OCR
Oxford Cambridge and RSA

Friday 19 May 2017 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Find $\sum_{r=1}^{n}(r^2-r-8)$, giving your answer in a fully factorised form. **[5]**

2 Use an algebraic method to find the square roots of the complex number $43-(6\sqrt{10})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[5]**

3 The matrices **A** and **B** are given by $\mathbf{A}=\begin{pmatrix}1 & 4\\ -2 & a\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}7 & 3\\ 1 & 5\end{pmatrix}$, where $a \neq -8$ and **I** is the 2×2 identity matrix. Find

(i) $7\mathbf{A}-\mathbf{I}$, **[2]**

(ii) $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}$. **[3]**

4 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}=\frac{n}{2n+1}$. **[5]**

5 The matrix $\begin{pmatrix}1 & 5\\ 0 & 1\end{pmatrix}$ represents the transformation P.

(i) Describe fully the transformation P. **[3]**

Transformation Q is a stretch, parallel to the y-axis with scale factor 4.

(ii) Find the matrix that represents transformation Q. **[2]**

Transformation T is equivalent to transformation P followed by transformation Q.

(iii) Find the matrix that represents transformation T. **[2]**

(iv) Find the area of the image of the unit square under transformation T. **[2]**

6 The complex number z_1 has modulus 3 and argument $\frac{3}{5}\pi$. The complex number z_2 has modulus 3 and argument $-\frac{9}{10}\pi$.

(i) Sketch on a single argand diagram z_1, z_2 and z_1-z_2. **[3]**

(ii) Find the exact value of $|z_1-z_2|$ and the exact value of $\arg(z_1-z_2)$. **[5]**

(iii) Give a geometrical description of the locus given by $|z-z_1|=|z-z_2|$. **[2]**

7 **(i)** Show that $\frac{1}{2r-1}-\frac{1}{2r+5}\equiv\frac{6}{(2r-1)(2r+5)}$. **[1]**

Hence find

(ii) $\sum_{r=2}^{30}\frac{6}{(2r-1)(2r+5)}$, giving your answer correct to 3 decimal places, **[5]**

(iii) $\sum_{r=2}^{\infty}\frac{6}{(2r-1)(2r+5)}$, giving your answer as a single fraction. **[1]**

8 In the cubic equation $4z^3+az^2+bz+c=0$, a, b and c are real numbers. One root is $1+\frac{3}{2}\mathrm{i}$ and the sum of the roots is 6. Find the values of a, b and c. **[7]**

9 The matrix **C** is given by $\mathbf{C}=\begin{pmatrix}a & 1 & 1\\ 3 & a & 1\\ 5 & 3 & 2\end{pmatrix}$.

(i) Find the value of a for which **C** is singular. **[5]**

In the three simultaneous equations given below, p is a constant.

$$\begin{aligned} ax+\ y+\ z&=p\\ 3x+ay+\ z&=p-1\\ 5x+3y+2z&=p-2\end{aligned}$$

(ii) Write down one value of a for which these equations have a unique solution, giving a brief reason. **[1]**

(iii) Using the value of a found in **(i)**, find the value of p for which these equations are consistent. **[3]**

10 The complex number $a+\mathrm{i}b$ is denoted by z and the complex number $c+\mathrm{i}d$ is denoted by w.

It is given that $z^2=z^*w$.

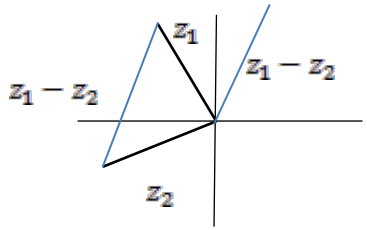
(i) Show that $2ab=ad-bc$. **[4]**

(ii) Given that the real part of $w=0$, find the values of b in terms of a. **[6]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance
1.			$\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-8n$	M1	Use at least 2 correct standard results, must have 3 terms
				A1	Obtain correct unsimplified answer
				M1	Attempt to factorise, must get at least a factor of n and simplify
			$\frac{1}{6}n(2n^2-50)$	A1	Obtain correct answer
			$\frac{1}{3}n(n-5)(n+5)$	A1 **[5]**	Obtain correct final answer
2.			$x^2-y^2=43, 2xyi=-6\sqrt{10}\,i$	M1	Attempt to equate real and imaginary parts of $(x+iy)^2$ and $43-6\sqrt{10}i$
				A1	Obtain both results a.e.f.
				M1	Obtain and solve a quadratic in x^2 or y^2 and attempt to square root $[\ x^4-43x^2-90=0, y^4+43y^2-90=0\ ]$
			$3\sqrt{5}-i\sqrt{2}, \quad -3\sqrt{5}+i\sqrt{2}$ Or $\pm(3\sqrt{5}-i\sqrt{2})$ or $\pm 3\sqrt{5}\mp i\sqrt{2}$	A1A1 **[5]**	Obtain correct answers as complex numbers a.e.e.f. **N.B.** $\pm(3\sqrt{5}\pm i\sqrt{2})$ gets A1, $\pm(3\sqrt{5}\mp i\sqrt{2})$ gets A1, $\pm 3\sqrt{5}\pm i\sqrt{2}$ gets A0
3.	(i)		$7\mathbf{A}-\mathbf{I}=\begin{pmatrix}6 & 28\\ -14 & 7a-1\end{pmatrix}$	B2 **[2]**	B1 for 3 elements correct or B1 for 4 elements correct but brackets omitted
	(ii)		*Either* $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}=\mathbf{BA}$ $\begin{pmatrix}1 & 28+3a\\ -9 & 4+5a\end{pmatrix}$	B1 M1 A1 **[3]**	Stated or used Attempt at multiplication of **BA** or **AB**, 2 elements correct Obtain correct answer
			Or $\mathbf{A}^{-1}=\frac{1}{a+8}\begin{pmatrix}a & -4\\ 2 & 1\end{pmatrix}$, $\mathbf{B}^{-1}=\frac{1}{22}\begin{pmatrix}5 & -3\\ -1 & 7\end{pmatrix}$	B1	Both correct
			$\begin{pmatrix}1 & 28+3a\\ -9 & 4+5a\end{pmatrix}$	M1 A1	Attempt at multiplication of their $\mathbf{A}^{-1}\mathbf{B}^{-1}$ only, ignore dets, 2 elements correct Obtain correct answer

Question			Answer	Marks	Guidance
4.				B1	Show clearly that result is true when $n = 1$
			$\frac{n}{2n+1}+\frac{1}{(2n+1)(2n+3)}$	M1*	Add correct $(n + 1)$th term to given result
			$\frac{n(2n+3)+1}{(2n+1)(2n+3)}$	DM1	Express as a single fraction with a correct denominator
			$\frac{n+1}{2n+3}$	A1	Show correct factorisation and obtain correct simplified answer
				B1 **[5]**	Clear statement of induction conclusion, previous 4 marks must be earned. Must include somewhere "true for $n = 1$", "true for n implies true for $n + 1$", "true for all n "
5.	(i)			B1 B1 B1 **[3]**	Shear, must be shear (only) otherwise 0/3 x-axis invariant, allow parallel to or along x-axis, in x direction (not in x-axis) Image of e.g. (0, 1) is (5,1) or column vectors allow 0.197^c, 11.3^o, $\tan^{-1}(1/5)$ or the complement, ignore scale factor if all OK otherwise
	(ii)		$\begin{pmatrix}1 & 0\\0 & 4\end{pmatrix}$	B1 B1 **[2]**	Each column correct
	(iii)		$\begin{pmatrix}1 & 5\\0 & 4\end{pmatrix}$	M1 A1ft **[2]**	Multiply matrices in correct order, or consider image of columns of P under Q Obtain correct answer, ft their (ii)
	iv)		(Area =) 4	M1 A1ft **[2]**	Find the determinant of a relevant matrix Obtain correct answer, ft their (iii) N.B. it is possible to consider scale factor for each transformation or draw a diagram

Question			Answer	Marks	Guidance
6	(i)			B1 B1 B1ft **[3]**	z_1 shown in 2nd quadrant, as a point or line segment z_2 shown in 3rd quadrant, as a point or line segment $z_1 - z_2$ clearly shown as line segment (either position) or point in 1st quadrant, $\lvert z_1 - z_2 \rvert$ must be > 3 and arg($z_1 - z_2$) reasonably accurate, ft from incorrect z_1 or z_2, ignore scales, penalise wrong "vector" arrows if shown
	(ii)		$3\sqrt{2}$ $7\pi/20$	B1 M1 A1 M1 A1 **[5]**	Find $z_1Oz_2 = 90^0$, or equivalent right angle Use Pythagoras' or other trig method Obtain correct answer a.e.e.f. Find relevant angles Obtain correct answer **N.B.** if they give decimal values in z_1 and z_2, max of M1 M1 only
	(iii)		Perpendicular bisector of Z_1Z_2	B2 **[2]**	Clear statement of required straight line Allow B1 for (straight) line (not circle)
7.	(i)		$\frac{2r+5-(2r-1)}{(2r-1)(2r+5)}$ or $\frac{2r+5-2r+1}{(2r-1)(2r+5)}$ must be seen	B1 **[1]**	Derive **given** result correctly (or use complete partial fractions method) **N.B.** 1st expression could be 2 separate fractions
	(ii)		*Either* 0.629 *Or* $\sum_{2}^{30}\frac{1}{2r-1} = 1.68237..$ $\sum_{2}^{30}\frac{1}{2r+5} = 1.05383..$ 0.629	M1* A1 A1 DM1 A1 **[5]** M1 A1 A1 M1 A1	Express at least 2 terms as differences using (i), could start at $r = 1$ Obtain $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}$,obtain $-\frac{1}{61}, -\frac{1}{63}, -\frac{1}{65}$ these may be unsimplified Show correct cancelling (and subtraction of 1st term if appropriate) Obtain correct answer Attempt to find sum of 2 separate series, could start at $r = 1$ Obtain correct answers, must be at least 4 decimal places Subtract their values Obtain correct answer
	(iii)		71/105	B1 **[1]**	Obtain correct answer

Question	Answer	Marks	Guidance
8.	*Either* $1-\frac{5}{2}i$	B1	State or use other complex root
	4	B1	State or use real root
	$\sum\alpha = -\frac{a}{4}$	M1	Use correct symmetric function, must include -ve
	$a = -24$	A1	Obtain correct answer
	$\sum\alpha\beta = \frac{b}{4}$ or $\alpha\beta\gamma = -\frac{c}{4}$	M1	Use another correct symmetric function
	$b = 45, c = -52$	A1 A1	Obtain correct answers
		[7]	
	Or $1-\frac{5}{2}i$	B1	State or use other complex root
	4	B1	State or use real root
		M1	Express as product of 3 linear factors
		M1	Expand to obtain a cubic expression $4x^3$ ……
	$a = -24, b = 45, c = -52$	A1A1A1	Obtain correct answers
	Or		
	$\sum\alpha = -\frac{a}{4}$	M1	Use correct symmetric function, must include -ve
	$a = -24$	A1	Obtain correct answer
		M1	Substitute given complex root into the cubic equation and attempt to simplify, use their *a*
		M1	
	$b = 45$	A1	Use imaginary part
		M1	Obtain correct answer
	$c = -52$	A1	Use real part
			Obtain correct answer
	Or		
		B1	
	$256 + 16a + 4b + c = 0$	B1	State of use real root
		M1	Use this root in cubic equation
		M1	Substitute given complex root into the cubic equation and attempt to simplify
	$a = -24, b = 45, c = -52$	A1A1A1	Use real and imaginary parts to obtain 2 more equations for *a* and *b*
			Obtain correct answers

Question			Answer	Marks	Guidance
9	(i)		$2a^2 - 8a + 8$ $a = 2$	M1 M1 A1 M1 A1 **[5]**	Show correct process for det of a 3 × 3, condone sign errors (Cramer's rule is M2) Show correct processes for a 2 × 2 Obtain correct answer Attempt to solve det C = 0, must be a quadratic Obtain correct answer
	(ii)		Any $a \neq 2$, so **C** non-singular or C has an inverse or det **C** $\neq 0$ or det C > 0	B1ft **[1]**	Must be consistent with their (i)
	(iii)		e.g. $x + y = -1, x + y = p$ or $2p - 1 = p - 2$ $p = -1$	M1 A1 A1 **[3]**	Put $a = 2$ (their a from (i)) and attempt to eliminate x, y or z or use eqn1 +eqn2 = eqn3 Obtain a correct pair of equations e.g. in x and y or correct equation Obtain correct answer
10.	(i)		$2ab = ad - bc$	B1 M1 M1 A1 **[4]**	State or use $z^* = a - ib$ Attempt to expand both expressions Equate imaginary parts Obtain **given** answer **N.B.** ignore errors in real parts
	(ii)		*Either* $a^2 - b^2 = bd, ad = 2ab$ $b = \pm\frac{a}{\sqrt{3}}$ *Or* $a^2 - b^2 + 2ab\text{i} = ad\text{i} + \text{bd}$ $d = \frac{(a^2 - b^2 + 2ab\text{i})(b - \text{i}a)}{a^2 + b^2}$ $b = \pm\frac{a}{\sqrt{3}}$	B1 M1 A1 M1 A2 **[6]** B1 M1 A1 M1 A2	State or use $c = 0$ Equate real parts, c need not = 0 at this stage Obtain 2 correct equations a.e.f. Eliminate d Obtain correct answers a.e.f., A1 for 1 correct answer **S.C.** $a = \pm\sqrt{3}b$ gets A1 State or use $c = 0$ in given expression Rearrange to make d the subject and rationalise Obtain correct answer Equate imaginary part to 0 Obtain correct answers a.e.f., A1 for 1 correct answer **S.C.** $a = \pm\sqrt{3}b$ gets A1